Solution

Class 12 - Mathematics

2020-2021 - Paper-4

Part - A

1. We are given that,

   $f(x) = \sin x + \cos x, x \in \left[0, \frac{\pi}{2}\right]$

   $f'(x) = \cos x - \sin x$

   For f(x) to be increasing, we must have

   f '(x) > 0

   cos x - sinx > 0

   sinx < cosx

   $\Rightarrow \frac{\sin x}{\cos x} < 1$

   tan x < 1

   $\Rightarrow x \in \left[0, \frac{\pi}{4}\right]$

2. We are given that,

   $f(x) = ax + b, a > 0$

   Let $x_1, x_2 \in R$ and $x_1 > x_2$

   $\Rightarrow ax_1 > ax_2$ for some $a > 0$

   $\Rightarrow ax_1 + b > ax_2 + b$ for some b

   $\Rightarrow f(x_1) > f(x_2)$

   $\therefore f(x)$ is strictly increasing function of R

3. 1. for f(x) to be increasing, we must have

      $f'(x) > 0$

      $\Rightarrow \quad 6\left(x^2 - 3x + 2\right) > 0$

      $= x^2 - 3x + 2 > 0$

      $(x - 1)(x - 2) > 0$

      $\Rightarrow \quad x < 1$ or $x > 2$

      $\Rightarrow \quad x \in (-\infty, 1) \cup (2, \infty)$

      so f(x) is increasing on $(-\infty, 1) \cup (2, \infty)$

   2. for f(x) to be decreasing, we must have

      $f'(x) < 0$

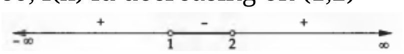
      $\Rightarrow \quad 6\left(x^2 - 3x + 2\right) < 0$

      $\Rightarrow \quad x^2 - 3x + 2 < 0$

      $\Rightarrow \quad (x - 1)(x - 2) < 0$

      $\Rightarrow \quad 1 < x < 2 \Rightarrow x \in (1, 2)$

      so, f(x) id decreasing on (1,2)

4. we have, f(x) = cos x + $cos\,(\sqrt{2}\,x)$

   using the inequality $|a + b| \le |a| + |b|$

   $|f(x)| = |cosx + cos\,(\sqrt{2}\,x)| \le |cos\,x| + |cos\,(\sqrt{2}\,x)| \le 1 + 1 = 2, \forall\, x\, inR$

   since $-1 \le \cos x \le 1 \Rightarrow |\cos x| \le 1$Which is true for any given angle.

   Hence, maximum value of f(x) = 2

5. $\int\left(6x^5-\frac{2}{x^4}-7x+\frac{3}{x}-5+4e^x+7^x\right)dx$

$=6\frac{x^{5+1}}{5+1}-2\frac{x^{-4+1}}{-4+1}-7\frac{x^2}{2}+3\ln|x|-5x+4e^x+\frac{7^x}{\ln 7}+c$

$=6\frac{x^6}{6}-2\frac{x^{-3}}{-3}-7\frac{x^2}{2}+3\ln|x|-5x+4e^x+\frac{7^x}{\ln 7}+c$

$=x^6+\frac{2}{3}x^{-3}-\frac{7}{2}x^2+3\ln|x|-5x+4e^x+\frac{7^x}{\ln 7}+c$,where c is constant of integration.

6. According to the question , $I=\int_1^2\frac{x^3-1}{x^2}dx$

$=\int_1^2\left(x-\frac{1}{x^2}\right)dx$

$=\left[\frac{x^2}{2}+\frac{1}{x}\right]_1^2$

$=\left(\frac{(2)^2}{2}+\frac{1}{(2)}\right)-\left(\frac{(1)^2}{2}+\frac{1}{(1)}\right)$

$=\left(2+\frac{1}{2}\right)-\left(\frac{1}{2}+1\right)$

$=1$

7. $\int\frac{2-3\sin x}{\cos^2 x}dx$

$=\int\left(\frac{2}{\cos^2 x}-\frac{3\sin x}{\cos^2 x}\right)dx$

$=\int 2\sec^2 x dx-3\int \tan x\sec x dx$

= 2tan x - 3sec x + C

8. $\left[-2\le x<-\frac{1}{2}\Rightarrow 2x+1<0\right]$ and $\left[-\frac{1}{2}\le x\le 1\Rightarrow 2x+1\ge 0\right]$

$\therefore\int_{-2}^{1}|2x+1|dx=\int_{-2}^{-1/2}|2x+1|dx+\int_{-1/2}^{1}|2x+1|dx$

$=\int_{-2}^{-1/2}-(2x+1)dx+\int_{-1/2}^{1}(2x+1)dx$

$=\left(-\frac{1}{4}+\frac{1}{2}\right)-(-4+2)+\left[2-\left(\frac{1}{4}-\frac{1}{2}\right)\right]$

$=\frac{1}{4}+2+\frac{9}{4}=\frac{9}{2}$.

9. $I=\int_0^1\tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

$I=\int_0^1\tan^{-1}\left(\frac{x+x-1}{1-x(x-1)}\right)dx$

$I=\int_0^1\left[\tan^{-1}(x)+\tan^{-1}(x-1)\right]dx$ ...(1)

$I=\int_0^1\left[\tan^{-1}(1-x)+\tan^{-1}(1-x-1)\right]dx[\because P_4]$

$I=\int_0^1\left[-\tan^{-1}(x-1)-\tan^{-1}(x)\right]dx$...(2) $[\because tan^{-1}(-\theta)=-tan^{-1}\theta]$

(1) + (2)

2I = 0

I = 0

10. We have, $y=x+\frac{4}{x^2}$.........(1)

$\Rightarrow y=x+4x^{-2}$

On differentiating w.r.t x, we get,

$\frac{dy}{dx} = 1 + 4 \times (-2x^{-3}) = 1 - 8x^{-3} = 1 - \frac{8}{x^3}$

$\frac{dy}{dx} = 1 - \frac{8}{x^3}$

Since the tangent is parallel to X-axis, therefore

$\frac{dy}{dx} = 0$

$\Rightarrow 1 - \frac{8}{x^3} = 0$

$\Rightarrow x^3 = 8$

$\Rightarrow x = 2$

From (1), when x = 2 , we get , y=$2 + \frac{4}{4} = 2 + 1 = 3$

Therefore, y=3 is required equation.

11. f(x) = $\frac{x}{\sin x}$ ...(1)

f'(x) = $\frac{\sin x - x \cdot \cos x}{\sin^2 x}$ [differentiate 1]

f'(x) = $\frac{\cos x(\tan x - x)}{\sin^2 x}$

in $[0, \frac{\pi}{2}]$ cosx > 0,

tan x - x > 0,

$\sin^2 x > 0$

hence f'(x) > 0,

So, function is increasing in the given interval.

12. I = $\int \frac{\left(\sin^{-1} x\right)^3}{\sqrt{1-x^2}} dx$

Put $\sin^{-1}x = t$

$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = dt$

$\therefore I = \int t^3 \cdot dt$

$= \frac{t^4}{4} + C$

$= \frac{\left(\sin^{-1} x\right)^4}{4} + C$ ($\because$ t = $\sin^{-1}$x)

13. Given f(x) = $\sin^7$ x.

$f(-x) = \sin^7(-x)$

$=[\sin(-x)]^7$

$=(-\sin x)^7$

$=-\sin^7 x$

= -f(x)

So f(x) is an odd function.

$\therefore \int_{-\pi/2}^{\pi/2} f(x)dx = 0 \Rightarrow \int_{-\pi/2}^{\pi/2} \sin^7 x dx = 0$

14. $I = \int \left(2^x + \frac{5}{x} - \frac{1}{x^{\frac{1}{3}}}\right) dx$

$=\int 2^x dx + \int \frac{5}{x} dx - \int \frac{1}{x^{\frac{1}{3}}} dx$

$=\int 2^x dx + 5\int \frac{1}{x} dx - \int x^{\frac{-1}{3}} dx$

$=\frac{2^x}{\log 2} + 5\log x - \frac{3}{2} x^{\frac{2}{3}} + c$

15. We know that $\int\sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$\int\sqrt{16 - 9x^2}dx = 3\int\{\sqrt{\left(\frac{16}{9} - x^2\right)}\}dx = 3\int\{\sqrt{\left(\frac{4}{3}\right)^2 - x^2}\}dx$

$= 3\left[\frac{x}{2}\sqrt{\frac{16}{9} - x^2} + \frac{8}{9}\sin^{-1}\frac{x}{(4/3)}\right] + C$

$= \frac{x}{2}\sqrt{16 - 9x^2} + \frac{8}{3}\sin^{-1}\left(\frac{3x}{4}\right) + C$

16. Formula = $\int\frac{1}{x}dx = \log x + c$

Therefore ,

Put x + log (sec x) = t

$\Rightarrow 1 + \frac{1}{\sec x}x\sec x \tan x\ dx = dt$

(1 + tan x)dx = dt

$\int\left(\frac{dt}{t}\right) = \int\frac{1}{t}dt = \log t + c$

= log (x + log (sec x)) + c

Section I

17. i. (a) $S = \pi r^2 + (\frac{600 + 2\pi r}{4})^2$

ii. (b) $r = \frac{300}{\pi + 4}$

iii. (c) $\frac{d^2s}{dr^2} > 0$

iv. (d) a = 2r

v. (a) $\frac{1}{2}$

18. i. (a) $-x^2 + 200x + 150000$

ii. (a) R'(x) = 0

iii. (b) Rs.100

iv. (d) 49

v. (c) 257, -63

Section II

19. Clearly, $9x^2 + 6x + 5 = (3x + 1)^2 + (2)^2$

$\Rightarrow \int\frac{1}{9x^2 + 6x + 5}dx = \int\frac{1}{(3x + 1)^2 + (2)^2}dx$

Let 3x + 1 = t

$\Rightarrow$ 3dx = dt

$\therefore \int\frac{1}{(3x + 1)^2 + (2)^2}dx = \frac{1}{3}\int\frac{1}{t^2 + 2^2}dt$

$= \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right] + C$

$= \frac{1}{6}\tan^{-1}\left(\frac{3x + 1}{2}\right) + C$

20. Let I = $\int\frac{e^{2x}}{e^{2x} - 2}dx$ ...(i)

Let $e^{2x} - 2 = t$ then,

$d(e^{2x} - 2) = dt$

$= 2e^{2x}dx = dt$

$\Rightarrow \quad dx = \frac{dt}{2e^{2x}}$

Putting $e^{2x} - 2 = t$ and $dx = \frac{dt}{2e^{2x}}$ in equation (i), we get,

$I = \int \frac{2e^{2x}}{t} \times \frac{dt}{2e^{2x}}$

$= \frac{1}{2}\int \frac{dt}{t}$

$= \frac{1}{2}\log|t| + c$

$= \frac{1}{2}\log|e^{2x} - 2| + c$ [$e^{2x} - 2 = t$]

$\therefore I = \frac{1}{2}\log|e^{2x} - 2| + c$

21. Let $I = \int_0^{\pi} \frac{x}{(1+\sin x)} dx$ ....(i)

Then, $I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin(\pi - x)} dx = \int_0^{\pi} \frac{(\pi - x)}{(1+\sin x)} dx$ ....(ii)

Adding (i) and (ii), we get

$2I = \pi\int_0^{\pi} \frac{dx}{(1+\sin x)} = \pi \cdot \int_0^{\pi} \frac{1}{(1+\sin x)} \times \frac{(1-\sin x)}{(1-\sin x)} dx$

or $2I = \pi\int_0^{\pi} \left(\frac{1-\sin x}{\cos^2 x}\right) dx = \pi \cdot \left[\int_0^{\pi} \sec^2 x dx - \int_0^{\pi} \sec x \tan x dx\right]$

$= \pi \cdot \left[[\tan x]_0^{\pi} - [\sec x]_0^{\pi}\right] = 2\pi$

$\therefore I = \pi$, i.e., $\int_0^{\pi} \frac{x}{(1+\sin x)} dx = \pi$

OR

Given that $f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$

$\Rightarrow f'(x) = 4 - \frac{1}{2}(2x) = 4 - x$

Now, f'(x) = 0

$\Rightarrow$ x = 4

Now, we evaluate the value of f at critical point x = 0 and at end points of the interval $\left[-2, \frac{9}{2}\right]$

$f(4) = 16 - \frac{1}{2}(16) = 16 - 8 = 8$

$f(-2) = -8 - \frac{1}{2}(4) = -8 - 2 = -10$

$f\left(\frac{9}{2}\right) = 4\left(\frac{9}{2}\right) - \frac{1}{2}\left(\frac{9}{2}\right)^2 = 18 - \frac{81}{8} = 18 - 10.125 = 7.875$

Therefore, the absolute maximum value of f on $\left[-2, \frac{9}{2}\right]$ is 8 occurring at x = 4

And, the absolute minimum value of f on $\left[-2, \frac{9}{2}\right]$ is -10 occurring at x = -2

22. Let, $I = \int (x+1)\sqrt{2x + x^2}dx$

Put, $(2x + x^2) = t$

$\Rightarrow$ (2 + 2x) dx = dt

$\Rightarrow$ 2dx(1+x) = dt

$\Rightarrow I = \frac{1}{2}\int t^{\frac{1}{2}} dt$

$\Rightarrow I = \frac{1}{2} \cdot \frac{2}{3} t^{\frac{3}{2}}$

$\Rightarrow I = \frac{1}{3}(2x + x^2)^{\frac{3}{2}}$

OR

Let $I = \int \frac{x^2+1}{(x-2)^2(x+3)} dx$

Using partial fractions,

$\frac{x^2+1}{(x-2)^2(x+3)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+3}$

$\Rightarrow\ x^2 + 1 = A(x - 2)(x + 3) + B(x + 3) + C(x - 2)^2$

$= (A + C)x^2 + (A + B - 4C)x + (-6A + 3B + 4C)$

Equating similar terms, we get

A + C = 1, A + B - 4C = 0, - 6A + 3B + 4C = 1

Solving, we get, $A = \frac{3}{5}$, B = 1, $C = \frac{2}{5}$

Thus,

$I = \frac{3}{5}\int\frac{dx}{x-2} + \int\frac{dx}{(x-2)^2} + \frac{2}{5}\int\frac{dx}{x+3}$

$I = \frac{3}{5}\log|x - 2| - \frac{1}{(x-2)} + \frac{2}{5}\log|x + 3| + c$

23. We have $\frac{x^2}{4} + \frac{y^2}{25} = 1$ ...(i)

Differentiating (i) with respect to x, we get

$\frac{x}{2} + \frac{2y}{25}\frac{dy}{dx} = 0$

or $\frac{dy}{dx} = \frac{-25}{4}\frac{x}{y}$

The tangent line is parallel to y-axis if the slope of the normal is 0, which gives $\frac{4y}{25x} = 0$, i.e., y = 0. Therefore, $\frac{x^2}{4} + \frac{y^2}{25} = 1$ for y = 0 gives x = $\pm$ 2. Hence, the points at which the tangents are parallel to the y-axis are (2, 0) and (–2, 0).

OR

Let $I = \int\frac{\csc^2 x}{\left(1-\cot^2 x\right)}dx$

Put t = cot x then dt = $-\text{cosec}^2 x dx$

$I = \int\frac{-dt}{\left(1-t^2\right)} = -\int\frac{1}{\left(1-t^2\right)}dt$

$= \frac{-1}{2}\log\left|\frac{1+\cot x}{1-\cot x}\right| + c$

24. Let $I = \int_0^{\frac{\pi}{4}}\sin^3 2t\cos 2t\,dt$.

Consider $\int\sin^3 2t\cos 2t\,dt$

Put sin 2t = u so that 2 cos 2t dt = du or cos 2t dt = $\frac{1}{2}$du

So $\int\sin^3 2t\cos 2t\,dt = \frac{1}{2}\int u^3 du$

$= \frac{1}{8}\left[u^4\right] = \frac{1}{8}\sin^4 2t = F(t)$

Therefore, by the second fundamental theorem of integral calculus

$I = F\left(\frac{\pi}{4}\right) - F(0) = \frac{1}{8}\left[\sin^4\frac{\pi}{2} - \sin^4 0\right] = \frac{1}{8}$

25. Let $I = \int\frac{\sin x - x\cos x}{x(x+\sin x)}dx$. Then,

$I = \int\frac{(x+\sin x) - x - x\cos x}{x(x+\sin x)}dx$

$\Rightarrow\quad I = \int\frac{(x+\sin x) - x(1+\cos x)}{x(x+\sin x)}dx$

$\Rightarrow\quad I = \int\left\{\frac{1}{x} - \frac{1+\cos x}{x+\sin x}\right\}dx$

$\Rightarrow\quad$ I = log |x| - log (x - sin x) + C

$\Rightarrow\quad I = \log\left|\frac{x}{x+\sin x}\right| + c$

26. Let $I = \int\sqrt{1 + e^x}e^x dx$ ...(i)

Let $1 + e^x = t$ then,

$d(1 + e^x) = dt$

$= e^x dx = dt$

$\Rightarrow \quad dx = \frac{dt}{e^x}$

Putting $1 + e^x = t$ and $dx = \frac{dt}{e^x}$, in equation (i), we get

$I = \int \sqrt{t} e^x \frac{dt}{e^x}$

$= \int t^{\frac{1}{2}} dt$

$= \frac{2}{3} \times \frac{t^{\frac{3}{2}}}{} + c$

$\therefore \quad = \frac{2}{3}\left(1 + e^x\right)^{\frac{3}{2}} + c$ [$1 + e^x = t$]

27. Given function is $\frac{x^3+x+1}{x^2-1}$

Dividing $(x^3 + x + 1)$ by $x^2 - 1$, we get,

$\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$

Let $\frac{2x+1}{x^2-1} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$

Now, $2x + 1 = A(x – 1) + B(x + 1)$ ...(i)

Substituting x = 1 and -1 in equation (i), we get,

$A = \frac{1}{2}$ and $B = \frac{3}{2}$

Thus, $\frac{x^3+x+1}{x^2-1} = x + \frac{1}{2(x+1)} + \frac{3}{2(x-1)}$

$\Rightarrow \int \frac{x^3+x+1}{x^2-1} = \int x dx + \frac{1}{2}\int \frac{1}{(x+1)} dx + \frac{3}{2}\int \frac{1}{(x-1)} dx$

$= \frac{x^2}{2} + \frac{1}{2}\log|x + 1| + \frac{3}{2}\log|x - 1| + C$

28. Let $I = \int \frac{1-\sin 2x}{x+\cos^2 x} dx$ ...(i)

Also let $x + \cos^2 x = t$ then, we have

$d(x + \cos^2 x) = dt$

$(1 - 2 \cos x \sin x)\, dx = dt$

$\Rightarrow \quad dx = \frac{dt}{1-2\cos x \sin x}$

Putting $x + \cos^2 x = t$ and $dx = \frac{dt}{1-2\cos x\sin x}$ in equation (i), we get

$I = \int \frac{1-\sin 2x}{t} \times \frac{dt}{1-2\cos x\sin x}$

$= \int \frac{1-\sin 2x}{t} \times \frac{dt}{1-\sin 2x}$

$= \int \frac{dt}{t}$

$= \log |t| + c$

$= \log |x + \cos^2 x| + c$

$\therefore \; I = \log |x + \cos^2 x| + c$

Section III

29. Let, $I = \int \frac{x^2}{x^4+x^2-2} dx$

Using partial fractions,

$\frac{x^2}{x^4+x^2-2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{x^2+2}$

$\Rightarrow \; x^2 = A(x - 1)(x^2 + 2) + B(x + 1)(x^2 + 2) + C(x^2 - 1)$

For, x = 1, $A = \frac{1}{6}$

For, x = -1, $B = -\frac{1}{6}$

For, x = 0, C = $-\frac{2}{3}$

$\because$ I = $\frac{1}{6}\int\frac{dx}{x+1} - \frac{1}{6}\int\frac{dx}{x-1} - \frac{2}{3}\int\frac{dx}{x^2+2}$

$\Rightarrow$ I = $\frac{1}{6}\log|x+1| - \frac{1}{6}\log|x-1| - \frac{2}{3\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$ + c

30. To solve this we use substitution

let $x = a\cos 2\theta$

Differentiating w.r.t. x, we get

$dx = -2a\sin 2\theta$

Now, $x = -a \Rightarrow \theta = \frac{\pi}{2}$

$x = a \Rightarrow \theta = 0$

$\therefore \int_{-a}^{a}\sqrt{\frac{a-x}{a+x}}dx = \int_{\frac{\pi}{2}}^{0}\sqrt{\frac{a(1-\cos 2\theta)}{a(1+\cos 2\theta)}}(-2\sin 2\theta)d\theta$

$= 2a\int_0^{\frac{\pi}{2}}\frac{\sin\theta}{\cos\theta}\cdot\sin 2\theta d\theta$ [$\because 1-\cos 2\theta = 2\sin^2\theta$., $1+\cos 2\theta = 2\cos^2\theta$ $-\int_a^b f(x)dx = \int_b^a f(x)dx$]

$= 2a\int_0^{\frac{\pi}{2}}\frac{\sin\theta\cdot 2\sin\theta\cos\theta}{\cos\theta}$

$= 4a\int_0^{\frac{\pi}{2}}\sin^2\theta d\theta$

$= 2a\int_0^{\frac{\pi}{2}}(1-\cos 2\theta)d\theta$

$= 2a\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= 2a\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= 2a\left[\frac{\pi}{2} - 0 - 0 + 0\right] = \pi a$

$\therefore \int_{-2}^{2}\sqrt{\frac{a-x}{a+x}}dx = \pi a$

31. Given, $I = \int\frac{x+2}{\sqrt{x^2+5x+6}}dx$...(i)

Let , $x + 2 = A\frac{d}{dx}\left(x^2+5x+6\right) + B$

$\Rightarrow x + 2 = A(2x+5) + B$ ...(ii)

Comparing the coefficients of x and constant terms on both sides we get ,

$\Rightarrow 2A = 1 \quad and \quad 5A + B = 2$

$\Rightarrow A = \frac{1}{2}$ and $B = -\frac{1}{2}$

from equation (i) and (ii) we get ,

$\therefore \quad I = \int\frac{\frac{1}{2}(2x+5) - \frac{1}{2}}{\sqrt{x^2+5x+6}}dx$

$= \frac{1}{2}\int\frac{2x+5}{\sqrt{x^2+5x+6}}dx - \frac{1}{2}\int\frac{1}{\sqrt{x^2+5x+6}}dx$

$I = \frac{1}{2}I_1 - \frac{1}{2}I_2$...(iii)

consider , $I_1 = \int\frac{2x+5}{\sqrt{x^2+5x+6}}dx$

$put\ \ x^2 + 5x + 6 = t \implies (2x+5)dx = dt$

$\therefore \quad I_1 = \int\frac{1}{\sqrt{t}}dt = 2\sqrt{t} + C_1$

$I_1 = 2\sqrt{x^2+5x+6} + C_1$ ...(iv)[$put\ \ t = x^2+5x+6$]

consider , $I_2 = \int \frac{1}{\sqrt{x^2+5x+6}} dx$

$= \int \frac{1}{\sqrt{x^2+2\times\frac{5}{2}\times x+6+\frac{25}{4}-\frac{25}{4}}} dx$

$= \int \frac{1}{\sqrt{\left(x+\frac{5}{2}\right)^2+6-\frac{25}{4}}} dx$

$= \int \frac{1}{\sqrt{\left(x+\frac{5}{2}\right)^2-\left(\frac{1}{2}\right)^2}} dx$

$= \log\left|\left(x+\frac{5}{2}\right)+\sqrt{\left(x+\frac{5}{2}\right)^2-\left(\frac{1}{2}\right)^2}\right| + C_2 \left[\because \int \frac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + C\right]$

$\Rightarrow \quad I_2 = \log\left|x+\frac{5}{2}+\sqrt{x^2+5x+6}\right| + C_2$...(v)

Putting the values of I_1 and I_2 from Equations (iv) and (v) in Equation (iii) we get ,

$\Rightarrow I = \frac{1}{2}\left[2\sqrt{x^2+5x+6}+C_1\right]-\frac{1}{2}\left[\log\left|x+\frac{5}{2}+\sqrt{x^2+5x+6}\right|+C_2\right]$

$= \sqrt{x^2+5x+6}+\frac{C_1}{2}-\frac{1}{2}\log\left|x+\frac{5}{2}+\sqrt{x^2+5x+6}\right|-\frac{C_2}{2}$

$= \sqrt{x^2+5x+6}+\frac{C_1}{2}-\frac{1}{2}\log\left|\frac{2x+5+2\sqrt{x^2+5x+6}}{2}\right|-\frac{C_2}{2}$

$= \sqrt{x^2+5x+6}+\frac{C_1}{2}-\frac{1}{2}\left[\log\left|2x+5+2\sqrt{x^2+5x+6}\right|-log(2)\right]-\frac{C_2}{2}$ $[\because log(\frac{m}{n}) = log(m)-log(n)]$

$= \sqrt{x^2+5x+6}+\frac{C_1}{2}-\frac{1}{2}\log\left|2x+5+2\sqrt{x^2+5x+6}\right|+\frac{1}{2}log(2)-\frac{C_2}{2}$ [$\because$ log 2 is a constant]

$\Rightarrow \quad I = \sqrt{x^2+5x+6}-\frac{1}{2}\log\left|2x+5+2\sqrt{x^2+5x+6}\right|+C$ [where, $C = \frac{C_1}{2}+\frac{1}{2}log(2)-\frac{C_2}{2}$]

OR

Let I = ∫ sin x sin 2x sin 3x. Then,

$I = \frac{1}{2}\int$ (2 sin 2x sin x) sin 3x dx

$\Rightarrow \quad I = \frac{1}{2}\int$ cos x - cos 3x) sin 3x dx

$\Rightarrow \quad I = \frac{1}{4}\int$ (2 sin 3x cos x - 2 sin 3x cos 3x) dx

$\Rightarrow \quad I = \frac{1}{4}\int$ (sin 4x + sin 2x - sin 6x) dx $= \frac{1}{4}\left\{-\frac{\cos 4x}{4}-\frac{\cos 2x}{2}+\frac{\cos 6x}{6}\right\}+C$

32. The equations of the two curves are

$y^2 = 4x$ .... (i)

and $x^2 = 4y$ .... (ii)

From (i), we obtain $x = \frac{y^2}{4}$.

Putting $x = \frac{y^2}{4}$ in (ii), we get

$\left(\frac{y^2}{4}\right)^2 = 4y \Rightarrow y^4 - 64y = 0$

$\Rightarrow$ y(y3 - 64) = 0 $\Rightarrow$ y(y3 - 64) = 0

$\Rightarrow$ y = 0, y = 4

From equation (i) when y = 0, we get the value x = 0

and again when y = 4, we get the value x = 4.
Thus the two curves intersect at the points (0, 0) and (4, 4).
Differentiating (i) with respect to x, we get
$2y\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$ .... (iii)
Differentiating (ii) with respect to x, we get
$2x = 4\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x}{2}$ ..... (iv)
Angle of Intersection at (0, 0): From equation (iii), we get

$$m_1 = \left(\frac{dy}{dx}\right)_{(0,0)} = \infty$$

Therefore, the tangent to curve (i) at (0,0) is parallel to y-axis.
From equation (iv), we get

$$m_2 = \left(\frac{dy}{dx}\right)_{(0,0)} = 0$$

Therefore, the tangent to curve (ii) at (0, 0) is parallel to x-axis.
Hence, the angle between the tangents to two curves at (0,0) is a right angle.
Consequently, the two curves intersect at right angle at (0, 0).
Angle of Intersection at (4, 4): From (iii), we obtain

$$m_1 = \left(\frac{dy}{dx}\right)_{(4,4)} = \frac{2}{4} = \frac{1}{2}$$

From (iv), we obtain

$$m_2 = \left(\frac{dy}{dx}\right)_{(4,4)} = \frac{4}{2} = 2$$

Let θ be the angle of intersection of the two curves. Then,

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2-(1/2)}{1+2\times(1/2)}\right| = \frac{3}{4}$$

OR

Volume, V = $\frac{1}{2}\pi l\left(\frac{D}{2}\right)^2$

$\Rightarrow$ V = $\frac{\pi D^2 l}{8}$

$\Rightarrow$ $l = \frac{8V}{\pi D^2}$...(1)

Total surface area = $\frac{\pi D^2}{4}$ + lD + $\frac{\pi Dl}{2}$

$\Rightarrow$ S = $\frac{\pi D^2}{4} + \frac{8V}{\pi D} + \frac{8V}{2D}$ ...[From eq. (1)]

$\Rightarrow$ $\frac{dS}{dD} = \frac{\pi D}{2} - \frac{8V}{\pi D^2} - \frac{8V}{2D^2}$

For maximum or minimum values of S, we must have

$\frac{dS}{dD} = 0$

$\Rightarrow$ $\frac{\pi D}{2} - \frac{8V}{\pi D^2} - \frac{8V}{2D^2} = 0$

$\Rightarrow$ $\frac{\pi D}{2} = \frac{8V}{D^2}\left(\frac{1}{\pi} + \frac{1}{2}\right)$

$\Rightarrow$ $D^3 = \frac{16V}{\pi}\left(\frac{1}{\pi} + \frac{1}{2}\right)$

Now,we have

$$\frac{d^2S}{dD^2} = \frac{\pi}{2} + \frac{16V}{D^3}\left(\frac{1}{\pi} + \frac{1}{2}\right)$$

$\Rightarrow \frac{d^2S}{dD^2} = \frac{\pi}{2} + \pi > 0$

$\Longrightarrow l = \frac{8V}{\pi D^2}$

$\Rightarrow l = \frac{8}{\pi D^2}\left[\frac{\pi D^3}{16}\left[\frac{2\pi}{\pi+2}\right]\right]$

$\Rightarrow l = D\left(\frac{\pi}{\pi+2}\right)$

$\Rightarrow \frac{l}{D} = \frac{\pi}{\pi+2}$

Hence proved.

33. Consider the integral

$I = \int \frac{x^2+1}{\left(x^2+4\right)\left(x^2+25\right)} dx$

Let $y = x^2$

Thus,

$\frac{x^2+1}{\left(x^2+4\right)\left(x^2+25\right)} = \frac{y+1}{(y+4)(y+25)}$

$\Rightarrow \frac{y+1}{(y+4)(y+25)} = \frac{A}{y+4} + \frac{B}{y+25}$

by using partial fraction

$\Rightarrow \frac{y+1}{(y+4)(y+25)} = \frac{A(y+25)+B(y+4)}{(y+4)(y+25)}$

$\Rightarrow$ y + 1 = Ay + 25A + By + 4B

Comparing the coefficients,we have,

A + B = 1 and 25A + 4B = 1

Solving the above equation, We have,

$A = \frac{-1}{7}$ and $B = \frac{8}{7}$

Thus using values of A, and B, we get, $I = \int \frac{x^2+1}{\left(x^2+4\right)\left(x^2+25\right)} dx$

$= \int \frac{\frac{-1}{7}}{x^2+4} dx + \int \frac{\frac{8}{7}}{x^2+25} dx$

$= \frac{-1}{7}\int \frac{1}{x^2+4} dx + \frac{8}{7}\int \frac{1}{x^2+25} dx$

$= \frac{-1}{7} \times \frac{1}{2}\tan^{-1}\frac{x}{2} + \frac{8}{7} \times \frac{1}{5}\tan^{-1}\frac{x}{5} + C$

$= \frac{-1}{14}\tan^{-1}\frac{x}{2} + \frac{8}{35}\tan^{-1}\frac{x}{5} + c$

34. We know that,

$\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$\frac{1}{5+3\cos x} = \frac{1}{5+3\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}$

$= \frac{1+\tan^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)+3\left(1-\tan^2\frac{x}{2}\right)}$

$= \frac{\sec^2\frac{x}{2}dx}{8+2\tan^2\frac{x}{2}}$

Therefore the given integral becomes

$\int_0^x \frac{dx}{5+3\cos x}dx = \frac{1}{2}\int_0^x \frac{\sec^2\frac{x}{2}}{2^2+\tan^2\frac{x}{2}}dx$

Let $\tan\frac{x}{2} = t$

Differentiating w.r.t. x, we get

$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$

Now,

$x = 0 \Rightarrow t = 0$

$x = \pi \Rightarrow t = \infty$

$\therefore \frac{1}{2}\int_0^x \left(\frac{\sec^2\frac{x}{2}dx}{2^2+\tan^2\frac{x}{2}}\right)dx$

$= \int_0^\infty \frac{dt}{2^2+t^2}$

$= \left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right]_0^\infty$

$= \frac{1}{2}\left[\tan^{-1}(\infty) - \tan^{-1}(0)\right]$

$= \frac{1}{2}\left[\tan^{-1}\left(\tan\frac{\pi}{2}\right) - \tan^{-1}(\tan 0)\right]$

$= \frac{1}{2}\left[\frac{\pi}{2} - 0\right]$

$= \frac{\pi}{4}$

$\therefore \int_0^x \frac{dx}{5+3\cos x}dx = \frac{\pi}{4}$

35. Let $I = \int \frac{4x^4+3}{(x^2+2)(x^2+3)(x^2+4)}dx$

Let $x^2 = y$

$\because \frac{4x^4+3}{(x^2+2)(x^2+3)(x^2+4)} = \frac{4y^2+3}{(y+2)(y+3)(y+4)}$

Now by using partial fraction.

Let $\frac{4y^2+3}{(y+2)(y+3)(y+4)} = \frac{4y^2+3}{(y+2)(y+3)(y+4)}$

$\Rightarrow 4y^2 + 3 = A(y+3)(y+4) + B(y+2)(y+4) + C(y+2)(y+3)$

For y = -2, $A = \frac{19}{2}$

For y = -3, B = -39

For y = -4, $C = \frac{67}{2}$

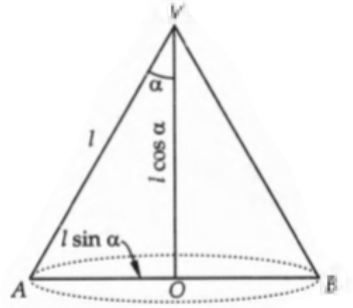
Thus, $I = \frac{19}{2}\int\frac{dx}{x^2+2} - 39\int\frac{dx}{x^2+3} + \frac{67}{2}\int\frac{dx}{x^2+4}$

$\Rightarrow I = \frac{19}{2\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{39}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + \frac{67}{4}\tan^{-1}\left(\frac{x}{2}\right) + c$

Section IV

36. Let α be the semi-vertical angle of a cone VAB of given slant height l.

In ΔAOV,



cosα = $\frac{VO}{VA}$ and sin α = $\frac{OA}{VA}$

$\Rightarrow$ cos α = $\frac{VO}{l}$ and sin α = $\frac{OA}{l}$

$\Rightarrow$ VO = l cos a, OA = l sin α

Let V be the volume of the cone, Then,

V = $\frac{1}{3}\pi$ (OA)^2 (VO)

$\Rightarrow$ V = $\frac{1}{3}\pi$ (l sin α)^2 (l cos α)

$\Rightarrow$ V = $\frac{1}{3}\pi l^3 \sin^2\alpha \cos \alpha$

$\Rightarrow$ $\frac{dV}{d\alpha} = \frac{\pi}{3} l^3$ (- $\sin^3 \alpha + 2 \sin \alpha \cos^2 \alpha$)

$\Rightarrow$ $\frac{dV}{d\alpha} = \frac{\pi l^3}{3} \sin \alpha (-\sin^2 \alpha + 2 \cos^2 \alpha)$ ....(i)

The critical points of V are given by $\frac{dV}{d\alpha}$ = 0.

$\because$ $\frac{dV}{d\alpha}$ = 0

$\Rightarrow$ $\frac{\pi l^3}{3} \sin \alpha (-\sin^2 \alpha + 2 \cos^2\alpha) = 0$

$\Rightarrow$ $2 \cos^2 \alpha = \sin^2 \alpha$

$\Rightarrow$ $\tan^2 \alpha = 2$ $\Rightarrow$ $\tan \alpha = \sqrt{2}$ [$\because$ a is acute $\because$ $\sin \alpha \neq 0$]

$\because$ $\cos \alpha = \frac{1}{\sqrt{1+\tan^2 \alpha}} = \frac{1}{\sqrt{3}}$[$\because$ $\tan \alpha = \sqrt{21}$]

Differentiating (i) with respect to a, we get

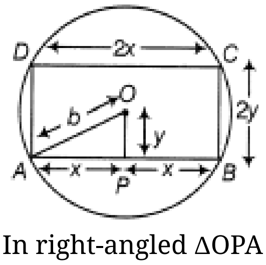
$\frac{d^2V}{d\alpha^2} = \frac{\pi l^3}{3}$ (- 3 $\sin^2 \alpha \cos \alpha + 2 \cos^3 \alpha$ -4 $\sin^2 \alpha \cos\alpha$) = $\frac{\pi l^3}{3} \cos^3 \alpha$ (2-7 $\tan^2 \alpha$)

$\because$ $\left(\frac{d^2V}{d\alpha^2}\right)_{\tan \alpha = \sqrt{2}} = \frac{1}{3}\pi l^3 \left(\frac{1}{\sqrt{3}}\right)^3 (2 - 7 \times 2) = \frac{-4\pi l^3}{3\sqrt{3}} < 0.$

Thus, V is maximum, when tan α= $\sqrt{2}$ or $\alpha = \tan^{-1}\sqrt{2}$ i.e. when the semi-vertical angle of the cone is $\tan^{-1}\sqrt{2}$.

OR

Let ABCD be the rectangle which is inscribed in a fixed circle whose centre is O and radius b. Let AB = 2x and BC = 2y.



In right-angled ΔOPA by Pythagoras theorem,
we have

$AP^2 + OP^2 = OA^2$

$\Rightarrow$ $x^2 + y^2 = b^2$

$\Rightarrow$ $y^2 = b^2 - x^2$

$\Rightarrow \quad y = \sqrt{b^2 - x^2}$ ...(i)

Let A be the area of the rectangle.

$\therefore$ A = (2x) (2y) [$\because$ area of rectangle = length $\times$ breadth]

$\Rightarrow \quad A = 4xy$

$\Rightarrow \quad A = 4x\sqrt{b^2 - x^2} \quad \left[\because \quad y = \sqrt{b^2 - x^2}\right]$

Therefore, on differentiating both sides w.r.t. x, we get,

$\frac{dA}{dx} = 4x \cdot \frac{d}{dx}\sqrt{b^2 - x^2} + \sqrt{b^2 - x^2} \cdot \frac{d}{dx}(4x)$ [by using product rule of derivative]

$\Rightarrow \frac{dA}{dx} = 4x \cdot \frac{-2x}{2\sqrt{b^2 - x^2}} + \sqrt{b^2 - x^2} \cdot 4$

$= 4\left[\frac{b^2 - x^2 - x^2}{\sqrt{b^2 - x^2}}\right]$

$\Rightarrow \frac{dA}{dx} = 4\left(\frac{b^2 - 2x^2}{\sqrt{b^2 - x^2}}\right)$

For maxima or minima, put $\frac{dA}{dx} = 0$

$\therefore \quad 4\left(\frac{b^2 - 2x^2}{\sqrt{b^2 - x^2}}\right) = 0$

$\Rightarrow \quad b^2 - 2x^2 = 0$

$\Rightarrow \quad 2x^2 = b^2$

$\Rightarrow \quad x = \frac{b}{\sqrt{2}}$ [$\because$ x cannot be negative]

Also, $\frac{d^2A}{dx^2} = \frac{d}{dx}\left(\frac{dA}{dx}\right) = \frac{d}{dx}\left[\frac{4\left(b^2 - 2x^2\right)}{\sqrt{b^2 - x^2}}\right]$

$\Rightarrow \quad \frac{d^2A}{dx^2} = \frac{d}{dx}\left[4\left(b^2 - 2x^2\right)\left(b^2 - x^2\right)^{-1/2}\right]$

$\Rightarrow \quad \frac{d^2A}{dx^2} = 4\left[-4x\left(b^2 - x^2\right)^{-1/2} + \left(b^2 - 2x^2\right)\left(-\frac{1}{2}\right)\left(b^2 - x^2\right)^{-3/2}(-2x)\right]$ [by using product rule of derivative]

$\Rightarrow \quad \frac{d^2A}{dx^2} = 4\left[\frac{-4x}{\sqrt{b^2 - x^2}} + \frac{x\left(b^2 - 2x^2\right)}{\left(b^2 - x^2\right)^{3/2}}\right]$

On putting $x = \frac{b}{\sqrt{2}}$, we get

$$\frac{d^2A}{dx^2} = 4\left[\frac{\frac{-4b}{\sqrt{2}}}{\sqrt{b^2 - \frac{b^2}{2}}} + \frac{\frac{b}{\sqrt{2}}\left(b^2 - 2\frac{b^2}{2}\right)}{\left(b^2 - \frac{b^2}{2}\right)^{3/2}}\right]$$

$$= 4\left[\frac{\frac{-4b}{\sqrt{2}}}{\sqrt{\frac{b^2}{2}}} + 0\right]$$

$\Rightarrow \quad \frac{d^2A}{dx^2} = -16 < 0$

$\therefore \frac{d^2A}{dx^2} < 0$. Therefore, A is maximum at $x = \frac{b}{\sqrt{2}}$

Now, on putting $x = \frac{b}{\sqrt{2}}$ in Eq. (i), we get

$$y = \sqrt{b^2 - \frac{b^2}{2}} = \sqrt{\frac{b^2}{2}} = \frac{b}{\sqrt{2}}$$

$$\therefore \quad x = y = \frac{b}{\sqrt{2}} \Rightarrow 2x = 2y = \sqrt{2}b$$

Thus, area of rectangle is maximum, when 2x = 2y, i.e. when rectangle is a square

37. Given: $\int_0^{\pi} x\ln(\sin x)dx$

Using the property $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

Let I = $\int_0^{\pi} x\ln(\sin x)dx$

$\Rightarrow \int_0^{\pi}(\pi - x)\ln(\sin(\pi - x))dx = \int_0^{\pi}\pi\ln(\sin x)dx - \int_0^{\pi}x\ln(\sin x)dx$

As sin (π - x) = sin x

$\Rightarrow$ 2I = $\int_0^{\pi}\pi\ln(\sin x)dx = \pi\int_0^{\pi}\ln(\sin x)dx$ ......(i)

Now in $\int_0^{\pi}\ln(\sin x)dx$

Using the property: $\int_0^{2a}f(x)dx = 2\int_0^{a}f(x)dx$ (for f(2a - x) = f(x))

$\Rightarrow \int_0^{\pi}\ln(\sin x)dx = 2\int_0^{\frac{\pi}{2}}\ln(\sin x)dx$ ......(ii)

Let Z = $\int_0^{\frac{\pi}{2}}\ln(\sin x)dx$ ......(iii)

Using the property $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

$$Z = \int_0^{\frac{\pi}{2}}\ln\left(\sin\left(\frac{\pi}{2} - x\right)\right)dx = \int_0^{\frac{\pi}{2}}\ln(\cos x)dx \text{ .....(iv)}$$

Adding equations (iii) and (iv),

$2Z = \int_0^{\frac{\pi}{2}}\ln(\sin x)dx + \int_0^{\frac{\pi}{2}}\ln(\cos x)dx = \int_0^{\frac{\pi}{2}}\ln(\sin x\cos x)dx$ .....(v)

$$\Rightarrow \int_0^{\frac{\pi}{2}}\ln(\sin x\cos x)dx = \int_0^{\frac{\pi}{2}}\ln\left(\frac{2\sin x\cos x}{2}\right)dx$$

$$\Rightarrow \int_0^{\frac{\pi}{2}}\ln\left(\frac{2\sin x\cos x}{2}\right)dx = \int_0^{\frac{\pi}{2}}(\ln(\sin 2x) - \ln 2)dx$$

$$\Rightarrow \int_0^{\frac{\pi}{2}}(\ln(\sin 2x))dx - \int_0^{\frac{\pi}{2}}(\ln 2)dx = \int_0^{\frac{\pi}{2}}\ln(\sin 2x)dx - \frac{\pi\ln 2}{2} \text{ ....(vi)}$$

Now in $\int_0^{\frac{\pi}{2}}\ln(\sin 2x)dx$ put 2x = t

$\Rightarrow$ 2dx = dt and limits changes from 0 to π

$$2Z = \frac{1}{2}\int_0^{\pi}\ln(\sin t)dt - \frac{\pi\ln 2}{2}$$

From equation (ii) $\frac{1}{2}\int_0^{\pi}\ln(\sin t)dt$ again becomes,

$$2Z = \frac{2}{2}\int_0^{\frac{\pi}{2}}\ln(\sin t)dt - \frac{\pi\ln 2}{2}$$

From equation (iii)

$$2Z = Z - \frac{\pi\ln 2}{2}$$

$$Z = \int_0^{\frac{\pi}{2}}\ln(\sin x)dx = -\frac{\pi\ln 2}{2} \text{ ......(vii)}$$

On putting (vii) in (ii) and the obtained result in(i)

$2I = -\pi^2\ln 2$

$$\Rightarrow I = \int_0^{\pi}x\ln(\sin x)dx = -\frac{\pi^2}{2}\ln 2$$

OR

According to the question , $I = \int_0^{\pi} \frac{x\tan x}{\sec x \cdot \operatorname{cosec} x} dx$ ....(i)

we know that , $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

we get

$I = \int_0^{\pi} \frac{(\pi - x)\tan\ (\pi - x)}{\sec\ (\pi - x)\operatorname{cosec}(\pi - x)} dx$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)(-\tan x)}{-\sec x\operatorname{cosec} x} dx$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)\tan x}{\sec x\operatorname{cosec} x} dx$......(ii)

On adding Equations (i) and (ii) we get,

$2I = \int_0^{\pi} \frac{\pi\tan x}{\sec x\operatorname{cosec} x} dx$

$\Rightarrow I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin^2 x(\cos x)}{(\cos x)} dx$

$= \frac{\pi}{2}\int_0^{\pi} \sin^2 x dx$

$= \frac{\pi}{2}\int_0^{\pi} \left(\frac{1-\cos 2x}{2}\right) dx$

$= \frac{\pi}{4}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi}$

$\Rightarrow I = \frac{\pi}{4}\left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin\ (0)}{2}\right]$

$\Rightarrow I = \frac{\pi}{4}[\pi - 0]$

$\Rightarrow I = \frac{\pi^2}{4}$

38. we will solve this as a limit of sum.

Here f(x) is continuous in [1, 3]

$\int_a^b f(x)dx = \lim_{n \to \infty} h\sum_{r=0}^{n-1}$ f(a + rh), where h = $\frac{(b-a)}{n}$

here h = $\frac{2}{n}$

$\int_1^3 \left(e^{-x}\right)dx = \lim_{n \to \infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} f\left(1 + \left(\frac{2r}{n}\right)\right)$

$= \lim_{n \to \infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} e^{-\left(1 + \frac{2r}{n}\right)}$

$= \lim_{n \to \infty}\left(\frac{2}{n}\right)\sum_{r=0}^{n-1} e^{-1} \cdot e^{-\frac{2r}{n}}$

Common ratio is h = $-\frac{2}{n}$

sum = e^{-1} ($e^0 + e^h + e^{2h} + \ldots + e^{nh}$)

$= \lim_{n \to \infty}\left(\frac{2e^{-1}}{n}\right) e^0 + e^h + e^{2h} + \ldots + e^{nh}$

sum of $e^0 + e^h + e^{2h} + \ldots + e^{nh}$

Whose sum is $= \frac{e^h\left(1 - e^{nh}\right)}{1 - e^h}$

$= \lim_{n \to \infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^h\left(1 - e^{nh}\right)}{1 - e^h}\right)$

$= \lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^{h}\left(1-e^{nh}\right)}{\frac{\frac{1-e^{h}.h}{h}}{}}\right.$

$\lim_{h\to 0}\frac{1-e^{h}}{h} = -1$

$= \lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^{h}\left(1-e^{nh}\right)}{-h}\right.$

As h = $-\frac{2}{n}$

$= \lim_{n\to\infty}\left(\frac{2e^{-1}}{n}\right)\left(\frac{e^{\left(-\frac{2}{n}\right)}\left(1-e^{n*(-2/n)}\right)}{2/n}\right.$

$= \frac{\left(1-e^{-2}\right)}{e}$

$= \frac{\left(e^{2}-1\right)}{e^{3}}$

OR

To find: Value of the given integral $\int\frac{(\sin x+\cos x)}{\sqrt{\sin 2x}}dx$

Formula to be used: $\int\frac{1}{\sqrt{1-x^{2}}}dx = \sin^{-1}x + c$

We have, $I = \int\frac{(\sin x+\cos x)}{\sqrt{\sin 2x}}dx$... (i)

Let (sinx – cosx) = t

$\Rightarrow (\cos x + \sin x) = \frac{dt}{dx}$

$\Rightarrow$ (cos x + sin x) dx=dt

$\Rightarrow$ $t^2 = \sin^2 x - 2\sin x.\cos x + \cos^2 x$

$\Rightarrow$ $t^2 = 1 - 2\sin x.\cos x$

$\Rightarrow$ $2\sin x.\cos x = 1 - t^2$

$\Rightarrow$ $\sin 2x = 1 - t^2$

Putting this value in equation (i)

$\Rightarrow I = \int\frac{dt}{\sqrt{1-t^{2}}}$

$I=\sin^{-1} t$

$I=\sin^{-1}(\sin x - \cos x)$

Let $\sin^{-1}(\sin x - \cos x)=\theta$

$\Rightarrow$ $I=\sin^{-1}(\sin x - \cos x)=\theta$... (ii)

$\Rightarrow$ $\sin\theta = \sin x - \cos x$

Now if $\sin\theta = \sin x - \cos x$

Then $\cos\theta = \sqrt{1-(\sin x-\cos x)^{2}}$

$\Rightarrow \cos\theta = \sqrt{1-\left(\sin^{2}x-2\sin x\cdot\cos x+\cos^{2}x\right)}$

$\Rightarrow \cos\theta = \sqrt{1-(1-2\sin x\cdot\cos x)}$

$\Rightarrow$ $\cos\theta = \sqrt{2\sin x\cdot\cos x)}$

Now $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Now tanθ = $\frac{\sin x - \cos x}{\sqrt{2\sin x \cos x)}}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\sin x - \cos x}{\sqrt{2\sin x \cdot \cos x)}}\right)$$

Comparing the value θ from eqn. (ii)

$$I = \theta = \tan^{-1}\left(\frac{\sin x - \cos x}{\sqrt{2\sin x \cdot \cos x)}}\right.$$

Dividing Numerator and denominator from cosx

$$I = \theta = \tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x)}}\right.$$

Hence the value.